TSUBAME 共同利用　令和3年度　学術利用　成果報告書

利用課題名　液体金属流れ CFD 手法の開発及び核融合研究への応用

英文：High-order flux reconstruction method for the hyperbolic formulation of the incompressible Navier-Stokes equations on unstructured grids

利用課題責任者
胡 長洪
所属
九州大学応用力学研究所 新エネルギー力学部門
https://www.tj.kyushu-u.ac.jp/

邦文抄録（300 字程度）

英文抄録（100 words 程度）

A high-order Flux reconstruction implementation of the hyperbolic formulation for the incompressible Navier-Stokes equation is presented. The governing equations employ Chorin's classical artificial compressibility (AC) formulation cast in hyperbolic form. Instead of splitting the second-order conservation law into two equations, one for the solution and another for the gradient, the Navier-Stokes equation is cast into a first-order hyperbolic system of equations. Including the gradients in the AC iterative process results in a significant improvement in accuracy for the pressure, velocity, and its gradients. Furthermore, this treatment allows for taking larger time-steps since the hyperbolic formulation eliminates the restriction due to diffusion.

Keywords: Hyperbolic method, Flux reconstruction, Unstructured grid, Incompressible Navier-Stokes equations, Artificial compressibility method

背景と目的

High order methods, such as flux reconstruction (FR) method, can achieve high order spatial accuracy on complicated geometries using compact stencils that only involve immediate face neighbors. Furthermore, high-order methods, such as the flux-reconstruction and discontinuous Galerkin methods have compact stencils which renders them particularly suitable for computation on modern hardware such as general purpose graphical processing units (GPGPUs). A challenge that arises when solving the Navier-Stokes equations using the classic formulation is the severe time-step restriction in diffusion dominated problems.

Even in advection dominated problems (i.e., high Reynolds number flows), localized high diffusion areas, either due to a high turbulent eddy viscosity or an artificially introduced stabilization viscosity, can have an adverse effect on stability, especially if such regions overlapped with highly refined mesh zones. To address this limitation in the context of the flux reconstruction method for incompressible flows, a hyperbolic method [1] for solving the advection-diffusion method is implemented in the flux reconstruction code PyFR[2]. Using hyperbolic diffusion, the restriction on stability from diffusion can be significantly alleviated. Additionally, the hyperbolic method was found to increase the accuracy and order of accuracy of variables and their gradients. This can be a desirable feature in applications that require accurate gradients.

概要

This research aims to develop next generation CFD techniques for solving incompressible, free surface flows. Such phenomena are important in a variety of fields and applications such as flooding and tsunami simulations, applications in naval and marine engineering, and nuclear fusion applications (flow of liquid metal as plasma-facing material). This development is based on the high-order Flux Reconstruction method, which allows obtaining more accurate results while utilizing modern hardware more efficiently when compared to conventional CFD techniques. The Tsubame super-computer was used to carry carry out simulations for testing and validation of GPU-accelerated, high-order CFD code for solving free surface incompressible flows. The simulations aimed to test the accuracy and computational efficiency and scaling of the code for problems involving a large number of degrees of freedom.

結果および考察

Numerical tests of the hyperbolic incompressible flux reconstruction implementation showed significant reductions in the absolute error of the field variables and the gradient of the velocity have been demonstrated . Additionally, it has been shown that equal orders of accuracy can be obtained for both the field variables and velocity gradients. Numerical results suggest that the improvement in the order of accuracy of the velocity gradients lead to a matching improvement in the order of accuracy of pressure. Analysis shows that the time-step requirements are significantly relaxed when using the hyperbolic solver.

This leads to a considerable speed-up of convergence especially for diffusion dominated problems where the parabolic restriction can be quite severe.

The strong scaling performance of the developed solver has been shown to be superior to the existing INS-FR solver due to the extra communication required for the computation of the viscous fluxes in the latter.

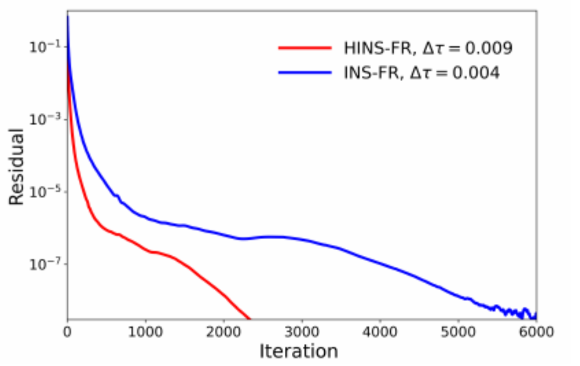

Figure 1: Convergence of the residual of the pressure for laminar flow past a sphere at Re=100 using the classic incompressibile NS (blue) and hyperbolic method (red)

Figure 1 shows the significant reduction in the number of iterations required by the hyperbolic solver to reach convergence, when compared to the classical incompressible solver.

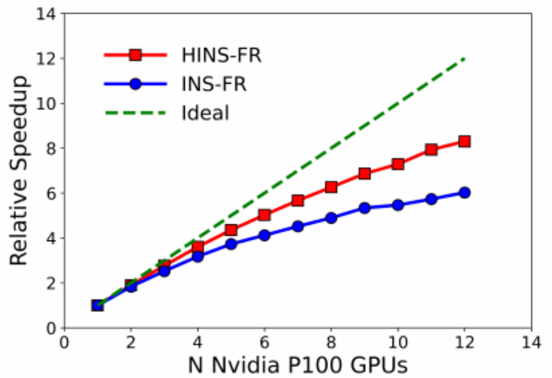

Figure 2: Strong scaling performance for the developed solver on multiple GPUs in TSUBAME

Figure 2 shows the result of a strong scaling study carried out on TSUBAME, demonstrating the better scaaling performance of the hyperbolic flux reconstruction method.

まとめ、今後の課題

In this report, progress on the development of a high-order hyperbolic method for incompressibloe flows for use in high-accuracy and large-scale simulations has been summarized. Satisfactory results of benchmark tests were obtained. Future work includes optimization of model parameters and reducing the memory foot-print and computational cost of the solver.

References

[1] H. Nishikawa, Y. Liu, Hyperbolic Navier-Stokes Method for HighReynolds-Number Boundary Layer Flows.

[2] Witherden FD, Farrington AM, Vincent PE. PyFR: An open source framework for solving advection–diffusion type problems on streaming architectures using the flux reconstruction approach. Computer Physics Communications. 2014 Nov 1;185(11):3028-40.